The Improbability Principle

why coincidences, miracles, and rare events happen every day

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- thinking of someone just before they phone you

- bumping into an old friend in a strange town

- meeting someone who has the same birthday as you

- finding that the number of your house keeps coming up in other situations

These sort of things make you wonder just what is going on

- Is there something we don't understand about the way the universe works?
- Do we misunderstand the laws of cause and effect?
- Is something exerting an invisible influence on our lives?

I’m going to argue that

- in a way the answer to these questions is ***yes***
- but that ***something*** is not a mystical supernatural force
- It is a consequence of mathematics and probability theory
- but looked at from an unusual perspective

The perspective of The Improbability Principle

extremely improbable events are commonplace

The improbability principle is not a single law, like $E = mc^2$

It's a set of five laws

1. ***The law of inevitability***
2. ***The law of truly large numbers***
3. ***The law of selection***
4. ***The law of the probability lever***
5. ***The law of near enough***

The law of inevitability:

Something must happen

A flipped coin is ***certain*** to come up

heads, tails, or ‘other’

It is ***certain*** that

one of the possible lottery tickets will come up

The case of the International Lotto Fund

- 15 Feb 1992 Virginia State Lottery rollover jackpot $27m
- A 6/44 lottery: there are only 7,059,052 sets of 6 numbers
- Each ticket has a 1 in 7,059,052 chance of winning
- Spend $7m to ***guarantee*** having the winning ticket

International Lotto Fund:

Consortium of 2,500 small investors

Australian, American, European, New Zealand

What could go wrong?

The logistics!
So complicated they managed to buy only 5m tickets
  $\Rightarrow$ Potential disaster: 1/4 chance of ***not*** buying the winning ticket

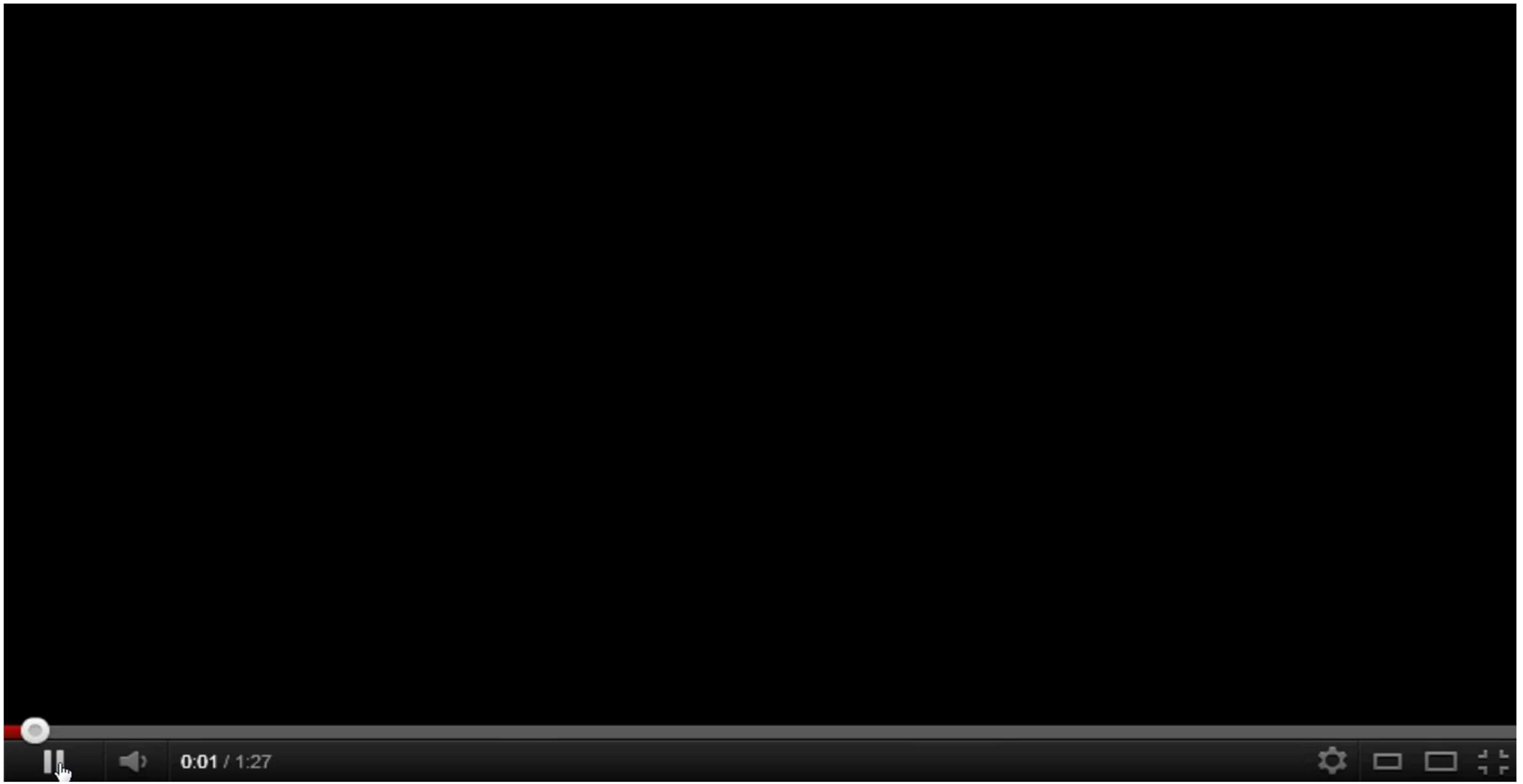
Also maybe they would have to ***share*** the jackpot with others

Legal challenge:
  each ticket must be paid for at the terminal which printed it

The law of truly large numbers:

with a large enough number of opportunities, any outrageous thing is likely to happen

- worldwide probability of any particular person being killed by a lightning strike in one year is very small: about 1 in 300,000
- but the world has c7bn people
- so the chance that no-one will be killed by lightning in one year is *infinitesimal*
- about 24,000 people are killed each year



https://www.google.co.uk/#q=man+struck+by+lightning+twice

The ***Tantilizer***, the *Bulgarian State lottery*:

6th September 2009 4, 15, 23, 24, 35, 42

10th September 2009 4, 15, 23, 24, 35, 42

Sports Minister ordered an investigation

But *the law of truly large numbers* says we should ***expect*** such things to happen

In a 6/42 lottery ***it's more likely than not*** that *some* two draws will match if two draws per week are made for 26 years [= 2704 draws]

Consider the number of lotteries around the world

- you'd ***expect*** to see duplicate sets of numbers drawn
- even on consecutive draws
- it would be ***amazing*** if we ***never*** saw such repeats

Israel's *Miphal HaPayis lottery*

same numbers on 21 Sept and 16 Oct 2010

North Carolina *Cash 5 lottery*

same numbers on 9 and 11 July 2007

The law of truly large numbers produces all sort of other lottery surprises

Maureen Wilcox bought tickets with the winning numbers for both the Massachusetts Lottery and the Rhode Island Lottery

Unfortunately, in each case the winning tickets were for the other lottery

The law of selection:

you can make things as likely as you want if you choose after the event

The arrows in the barn ("Texas sharpshooter")

Whole walnuts

Jeane Dixon

On 13th May 1956, four years before John Kennedy was elected, Ms Dixon published a prediction in *Parade Magazine* that the 1960 US Presidential election would be won be a Democrat who would then be assassinated or die in office

But Jeane Dixon also predicted

- that someone from the Soviet Union would be the first to walk on the moon
- that World War III would begin in 1958

The Jeane Dixon effect

- make enough predictions and you're sure to get some right
- tell people about those, and forget the wrong ones

The law of the probability lever:

slight changes can make highly improbable events almost certain

- c.100 people get killed by lightning in the US each year
- probability of getting killed by lightning in the US is about 1 in 3 million = $1/10^{th}$ worldwide probability

Because:

- US buildings are better protected than in many places
- people spend more time inside

Roy Sullivan: struck by lightning

- 1942: lost big toe nail
- 1969: lost eyebrows
- 1970: seared left shoulder
- 1972: hair set on fire
- 1973: regrown hair burnt
- 1976: ankle injured by lightning
- 1977: chest and stomach burned

How can this be explained ?

Roy Sullivan was a Virginia park ranger

The AstraZeneca (AZ) COVID-19 vaccine and cerebral venous thrombosis (CVT)

- March 15th 2021, 7 cases of CVT within 4 to 16 days of vaccination with the AZ vaccine in about 1.6 million vaccinations in Germany

- Background data show about 1 case expected in population of this size over a 14 day period

- Evidence that the AZ vaccine causes CVT?

Small probabilities are difficult to estimate accurately

Is the vaccinated group comparable with the population giving the estimated background rate?

- In Germany people over the age of 65 were not in the group to be initially vaccinated using the AZ vaccine
- Were potentially vulnerable people prioritised?
- Were health workers prioritised?
- Medication differences

A 12% difference in the estimate of the standard rate *doubles* the chance of 7 or more COVID-19 cases in 1.6 million people

The law of near enough:

events which are sufficiently similar are regarded as identical

1986 Bill Shaw survived a train crash that killed 13 people

2001 Ginny Shaw, Bill's wife, survived a train crash that killed 10 people

Rail travel is *safe*: 1 fatality per 10 billion passenger miles in UK

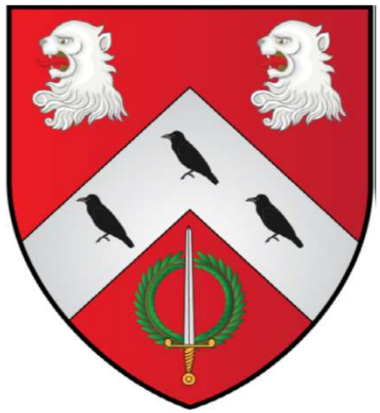
- Bill in both accidents?
- Accidents on same day?

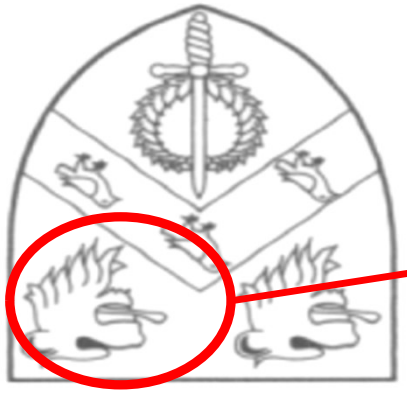
What counts as a lottery win?

The media often count lottery *near misses* as wins

- April 2007: Robert Hong “won” $340,000 by matching 5 out of 6 balls
- June 2011: Mike McDermott “won” £194,501 by matching 5 out of 6 numbers, plus the bonus ball
- April 2012: Virginia Pike “won” $1m with two tickets, each matching 5 out of 6 balls

St Anne’s College, Oxford.





Each law by itself can make apparently highly improbable events happen

But when the laws work together ...

Jeane Dixon:

- *the law of truly large numbers*
- *the law of selection*
- *the law of near enough*

How to make a fortune as a stock tipster*

- *the law of inevitability*
- *the law of selection*

** Please do not follow this advice!*

The reproducibility crisis in (some) sciences:

John Ioannidis:

" … most current published research findings are false."

- *The law of selection* - the file drawer effect
- *The law of truly large numbers* - p-values ignoring multiple endpoints, multiple groups, …
- *The law of the probability lever* – e.g. inappropriately assuming independence or particular distributions

The human mind

The conjunction fallacy

John initially took a degree in mathematics, and followed it with a PhD in astrophysics. After that, he worked in the physics department of a university for a while but then found a job in the back room of an algorithmic trading company, developing highly sophisticated statistical models for predicting movements of the financial markets. In his spare time he attends science fiction conventions.

Which has the higher probability?

A: John is married with two children

B: John is married with two children, and likes to spend his evenings tackling mathematical puzzles and playing computer games

Law of transposed conditional

Base rate fallacy

Possibility effect

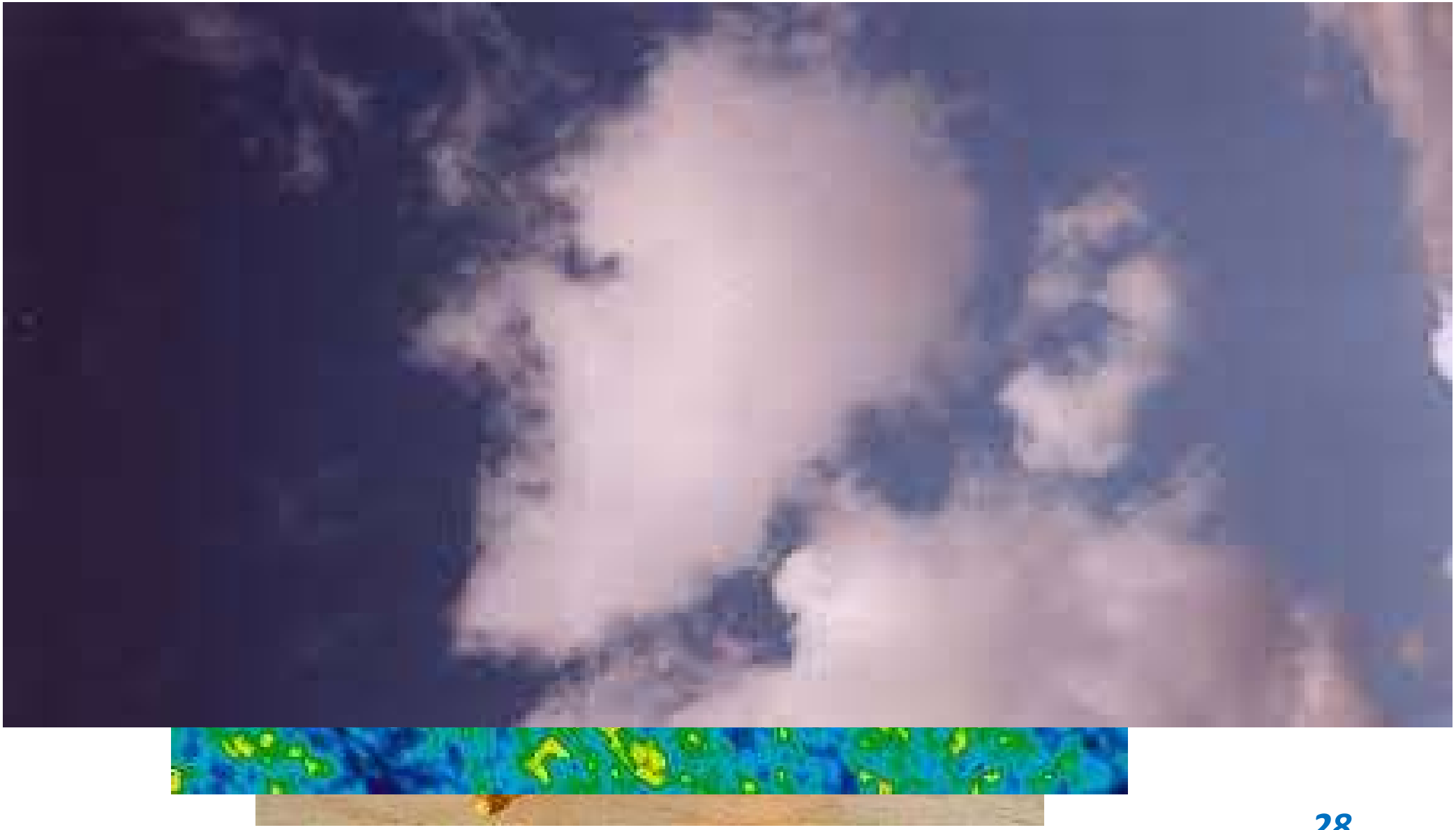
Hindsight bias

The law of small *numbers*

. . . . . .

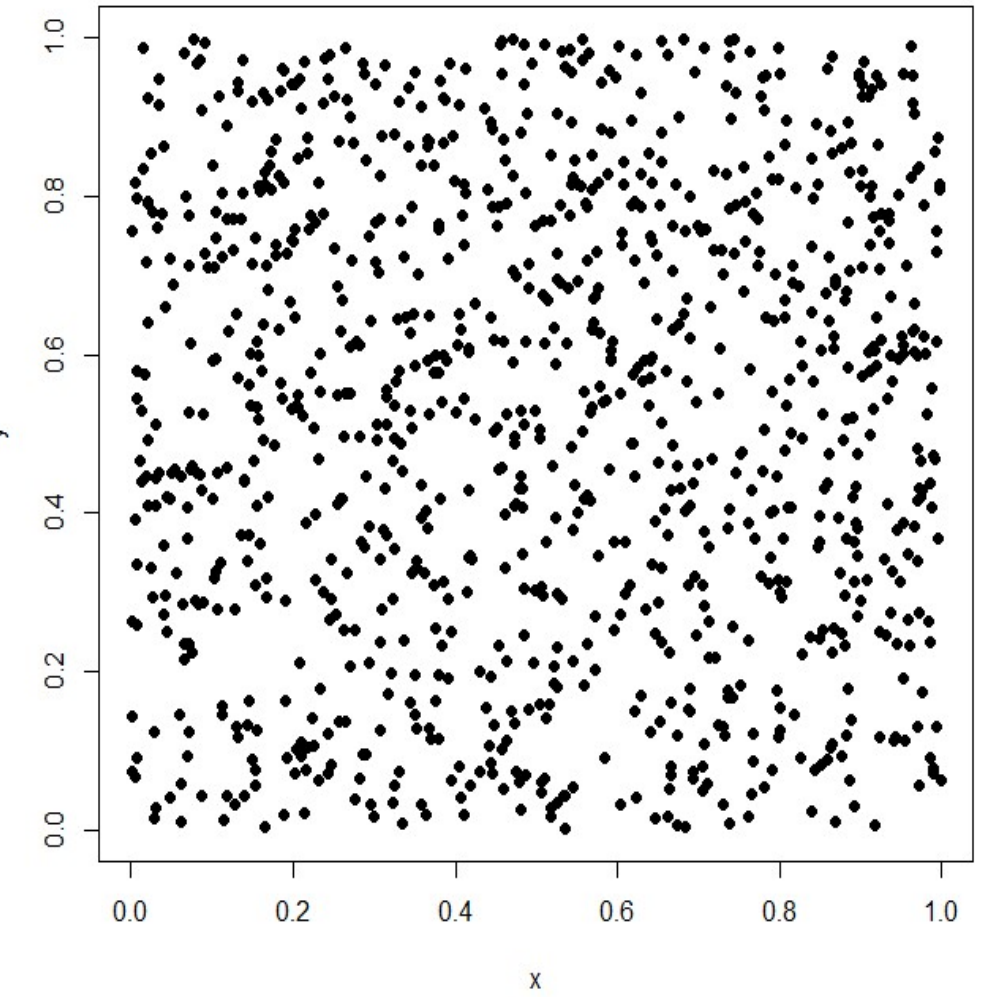
And, perhaps above all, pareidolia

Seeing patterns in random data



Law of truly large numbers:

- Search in many places
- Search for many kinds of pattern
  - local clusters
  - strings
  - voids

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Beyond the everyday

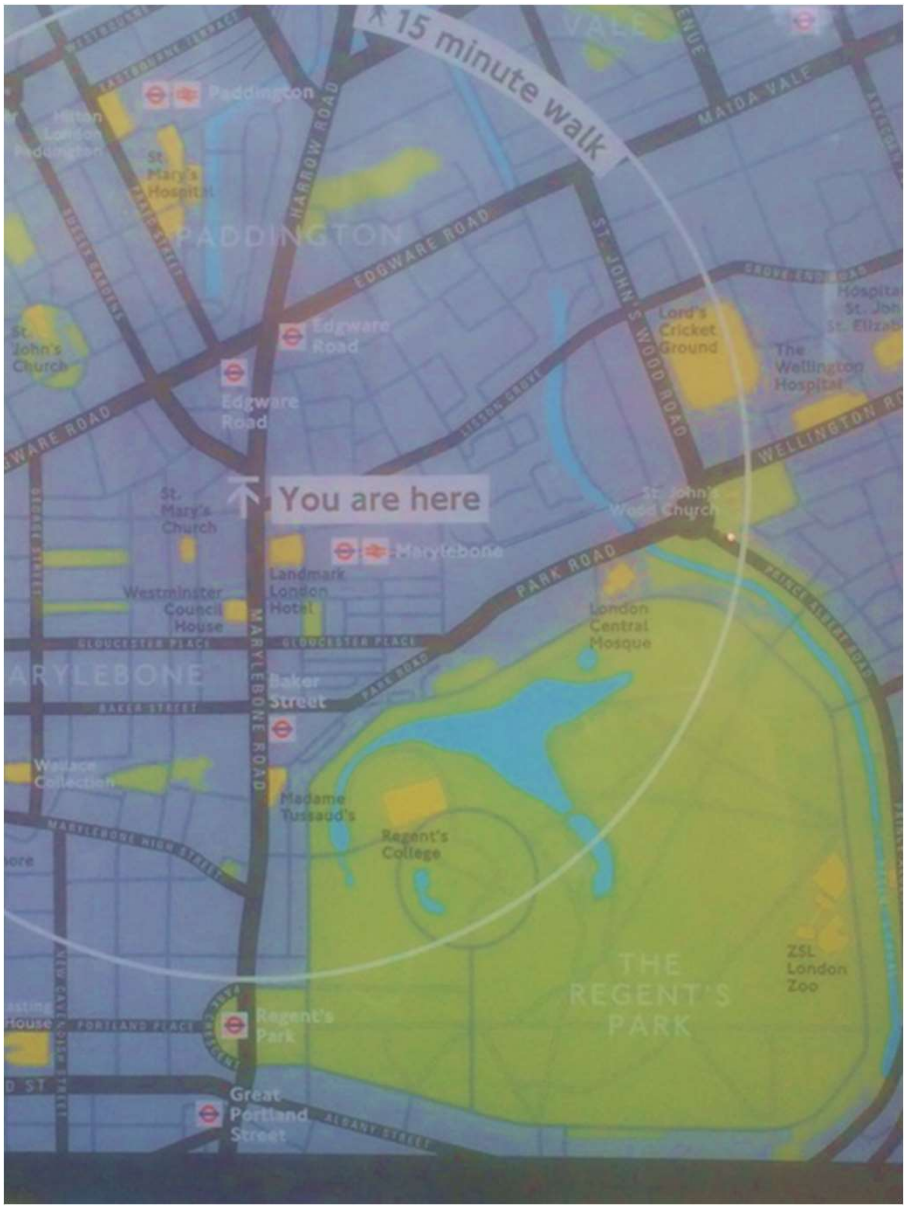
To life, the universe, and everything

Evolution:

- the law of truly large numbers
- the law of selection

The universe and the anthropic principle

- the ultimate instance of the law of selection


15 minute walk
You are here
Paddington
St Mary's Hospital
PADDINGTON
EDGWARE ROAD
HARROW ROAD
MAIDA VALE
ST JOHN'S WOOD ROAD
Edgware Road
St John's Church
Lord's Cricket Ground
The Wellington Hospital
St Mary's Church
Marylebone
Landmark London Hotel
Westminster Council House
PARK ROAD
London Central Mosque
MARYLEBONE
Baker Street
BAKER STREET
MARYLEBONE ROAD
Madame Tussaud's
Regent's College
THE REGENT'S PARK
ZSL London Zoo
Regent's Park
PORTLAND PLACE
Great Portland Street
GLOUCESTER PLACE

Using the improbability principle

Balancing probabilities
Bending probabilities
Believing probabilities

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Choose a die and
throw it 100 times

All 100 show 6

Do you think the die is
white or black?

- MEMO -

To the Members of the California State Assembly:

I am returning Assembly Bill 1176 without my signature.

For some time now I have lamented the fact that major issues are overlooked while many unnecessary bills come to me for consideration. Water reform, prison reform, and health care are major issues my Administration has brought to the table, but the Legislature just kicks the can down the alley.

Yet another legislative year has come and gone without the major reforms Californians overwhelmingly deserve. In light of this, and after careful consideration, I believe it is unnecessary to sign this measure at this time.

Sincerely,
Arnold Schwarzenegger

Stark, 2010, Chance, 23, p43-46

Using the improbability principle

Balancing probabilities

Bending probabilities

Believing probabilities

Buy one 6/49 lottery ticket per week for 20 years = 52×20 tickets

Prob jackpot = 0.0000743688

Or buy 52×20 (different) tickets in one week, at same cost

Prob jackpot = 0.0000743716

52×20 *different* tickets!

Except: allow for The Improbability Principle:

In April 2013 Harry Black bought two British Columbia Lottery tickets with the same numbers,

and won

Using the improbability principle

Balancing probabilities

Bending probabilities

Believing probabilities

Why movie sequels are less likely to be big hits than the originals

Why US TV programs are better when viewed in the UK; and UK programs are better when viewed in the US

Why the gambler's fallacy is wrong

Why extremely improbable events are commonplace

- ***The law of inevitability***
- ***The law of truly large numbers***
- ***The law of selection***
- ***The law of the probability lever***
- ***The law of near enough***
- ***And***

  the human mind

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